

PAM3012

Digital Image Processing for Radiographers

Mathematical Definitions

In this lecture

- Function
- Function Notation
- Image Functions
- Rates of Change
- Sum Function

Aim

Develop mathematical language to describe images quantitatively

What is a Function?

- A mathematical function is a process that converts one set of numbers into another.
- For example: Doubling

Doubling Function	
Input (x)	Output (y)
1	2
2	4
3	6
4	8

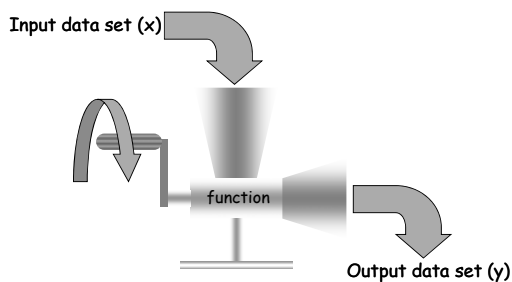
$$y = 2 \times x$$

or

$$y = 2x$$

- Important: For each input, there is only one possible output!

The Function Machine



Function Notation

- Variables
 - Dependent Variable (y)
 - Independent Variable (x)
- Constants

$$y = x + 2$$

$$f(x) = x + 2$$

Examples of Functions

- Circumference of a circle:
 - Circumference = $2 \times \pi \times \text{radius}$
 - $f(r) = 2\pi r$
- Area of a circle:
 - Area = $\pi \times \text{radius}^2$
 - $f(r) = \pi r^2$
- Volume of a sphere
 - Volume = $\frac{4}{3} \pi \times \text{radius}^3$
 - $f(r) = \frac{4}{3} \pi r^3$

Functions

- Functions can have more than one independent variable
- E.g. 2D function

$$a = b + c$$

$$f(b,c) = b + c$$

Image Function

- Digital Image can be described by a discrete 2D function
- x, y are pixel coordinates
- $f(x, y)$ is the gray level in each pixel

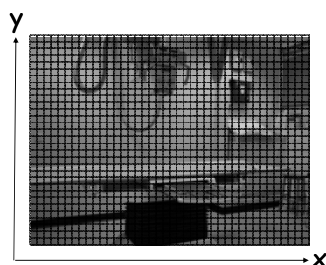


Image Function

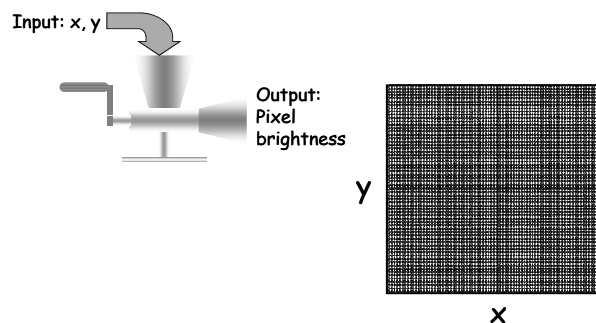
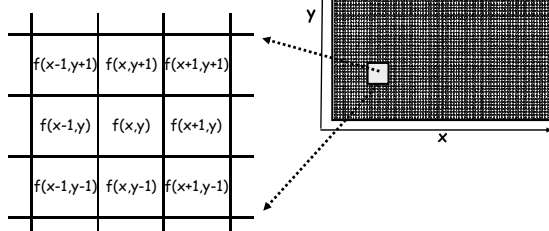


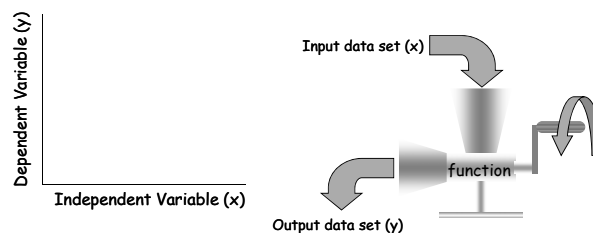
Image Function

- Pixel Notation



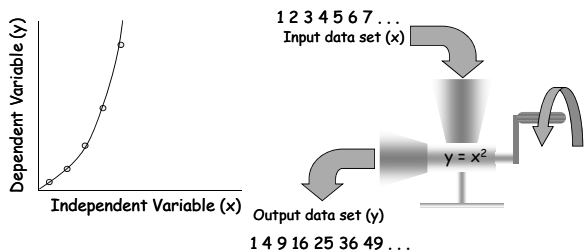
Graphs of functions

- Functions generate pairs of numbers
 - These can be used as co-ordinates to draw graphs
 - Graphical display of function

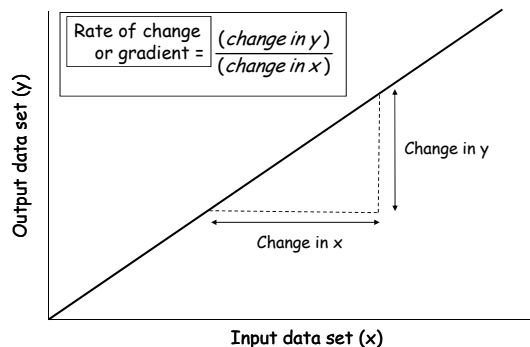


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Rates of Change

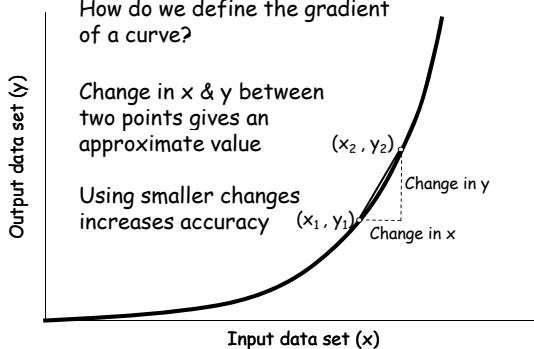


Rates of Change

How do we define the gradient of a curve?

Change in x & y between two points gives an approximate value

Using smaller changes increases accuracy



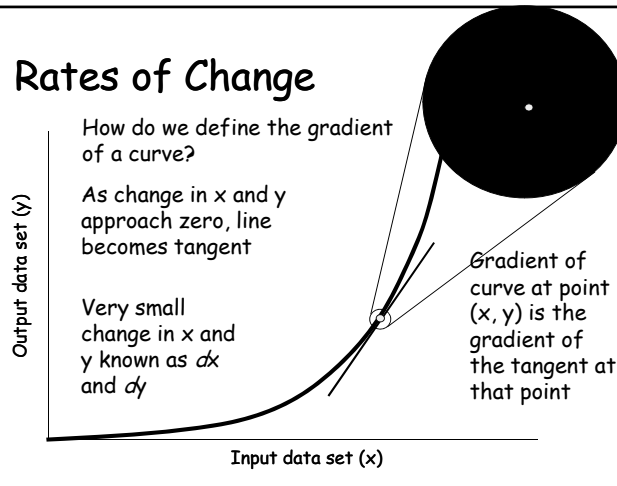
Rates of Change

How do we define the gradient of a curve?

As change in x and y approach zero, line becomes tangent

Very small change in x and y known as dx and dy

Gradient of curve at point (x, y) is the gradient of the tangent at that point

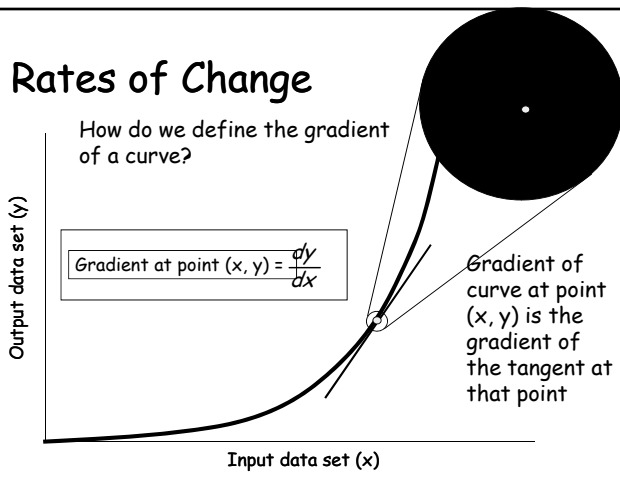


Rates of Change

How do we define the gradient of a curve?

$$\text{Gradient at point } (x, y) = \frac{dy}{dx}$$

Gradient of curve at point (x, y) is the gradient of the tangent at that point



Rates of Change

1D function $f(x)$

$$\text{Rate of change or gradient} = \frac{\text{change in } f(x)}{\text{change in } x}$$

$$\frac{df}{dx} = f(x+1) - f(x)$$

